K.L.E. Society's

Raja Lakhamagouda Science Institute

(Autonomous)

BELAGAVI.

SUBJECT

M.Sc - I Semester

Dec - 2018

(Mathematics)

QUESTION PAPER BOOKLET

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Reg.No. | | | | | | | |

KLE Society's

Raja Lakhamagouda Science Institute (Autonomous),

Belagavi.

First Semester M.Sc. Degree Examination Dec- 2018

AM01: TOPOLOGY

Duration: **3 Hrs** Max Marks: **70**

Instructions to candidates:

1) Attempt any Five Questions.
2) Figures to the right indicate full marks

1. a) Define topological space and give an example. Prove that in any topological space (X, τ) a subset G is open iff it is a neighborhood of each of its points.

b) Define Limit point and derived set. Give an example for each.
Prove that a subset A of a topological space (X, τ) is closed iff $D(A) \subseteq A$.

[7+7]

2. a) Define base for a topology. Let (X, τ) be a non-empty set and ß be a collection of subsets of X. Prove that ß is a base for topology on X, iff ß has the following properties.

(i) $X = \bigcup_{B \in ß} ß$

(ii) For any $B_1, B_2 \in ß$, the set $B_1 \cap B_2$ is a union of members of ß.

b) Let (X, τ) and (Y, σ) be any two topological spaces. Prove that a function $f: X \rightarrow Y$ is continuous iff for every subset B of Y, $f^{-1}(B^0) \subseteq [f^{-1}(B)]^0$

[8+6]

3. a) Prove that every second countable space is separable. Show that converse is not true.

b) Define a Lindelof space and give an example. Prove that every closed subspace of a Lindelof space is Lindelof. **[8+6]**

4. a) Let (X, τ) be a topological space and E is a connected subset of X. Let $F \subseteq X$ such that $E \subseteq F \subseteq \overline{E}$. Then prove that F is connected in particular $\overline{E}$ is connected.

b) Show that the space (R, U) is connected.

c) Define locally connected space. Prove that every discrete space is locally connected.

[5+5+4]

5. a) Define compactness in topological space. Prove that every closed subset of compact space is compact.

b) Show that every compact topological space has a Bolzano Weirstrass Property.

c) Define locally compact space. Prove that every compact space is locally compact.

[5+5+4]

Important Note: 1. On completing answers, compulsorily draw diagonal lines on the remaining blank pages.
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 PTO

6. **a)** Prove that a topological space (X, τ) is T_1-space iff every singleton subset of X is closed in X.

b) Prove that the property of being T_2-space is preserved under one-one, on to and open mappings.

c) Prove that every finite T_1-space is finite.

[6+5+3]

7. **a)** Define regular space and give an example. Prove that every compact Hausdorff space is normal and hence T_4-space.

b) Define completely regular space. Prove that every completely regular space is regular.

c) Prove that every discrete space is T_4-space.

[6+5+3]

8. **a)** Define a product space XxY on two topological spaces X and Y. Then prove that the projection π_x and π_y of XxY on X and Y are continuous and open mappings.

b) Let (X, τ) and (Y, σ) be two topological spaces. Prove that a product space XxY is compact iff X and Y are compact.

[7+7]

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Raja Lakhamagouda Science Institute (Autonomous), Belagavi.

First Semester M.Sc. Degree Examination Dec – 2018

AM02: LINEAR ALGEBRA

Duration: 3 Hrs **Max Marks: 70**

Instructions to candidates:

1) Attempt any Five Questions.
2) All questions carry equal marks.
3) Figures to the right indicate full marks.

1. a) Define a linear span L(S) of a subset S of a vector space V(F). Prove that the linear span L(S) of any subset S of a vector space V(F) is a subspace of V generated by S.

b) If S, T are subsets of a vector space V(F) then prove that

(i) $S \subseteq T$ implies that $L(S) \subseteq L(T)$ and **(ii)** $L(S \cup T) = L(S) + L(T)$.

c) In $V_3(R)$ where R is the field of real numbers, examine each of the following sets for linear dependence.

i. {(1, 2, 0), (0, 3, 1), (-1, 0, 1)}

ii. {(-1, 2, 1), (3, 0, -1), (-5, 4, 3)}. **[5+5+4]**

2. a) Prove that any two finite dimensional isomorphic vector space have the same dimensions.

b) If W_1 and W_2 are subspaces of finite dimensional vector space V(F) then prove that $\dim (W_1+W_2) = \dim W_1 + \dim W_2 - \dim (W_1 \cap W_2)$. **[6+8]**

3. a) State and prove rank nullity theorem.

b) Let U(F) be a finite dimensional vector space of dimension n and $B = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ be a basis of U. Let V(F) vector space and $\beta_1, \beta_2, \ldots, \beta_n$ be any n vectors in V. Then prove that there exists a unique linear transformation $T: U \rightarrow V$ such that $T(\alpha_i) = \beta_i$, i=1,2,…,n. **[8+6]**

4. a) Let V(F) be a finite dimensional vector space and W be a subspace of V. Prove that

i) dim W+dim A(W)=dim V and

ii) A(A(W))=W.

b) If $\lambda_1, \lambda_2, \ldots, \lambda_k$ in F are distinct characteristic roots of T in A(V) where V(F) is a vector space and $v_1, v_2, v_3, \ldots, v_k$ are characteristic vectors of T belonging to $\lambda_1, \lambda_2, \ldots, \lambda_k$ respectively then prove that $v_1, v_2, v_3, \ldots, v_k$ are linearly independent.

[8+6]

Important Note: 1. On completing answers, compulsorily draw diagonal lines on the remaining blank pages.
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PTO

5. **a)** Let V(F) be a finite dimensional vector space. If $\lambda \in F$ is a characteristic root of $T \in A(V)$ then prove that for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristic root of $q(T)$.

b) If V(F) is a finite dimensional vector space of dimension n and if $T \in A(V)$ has all its characteristic roots in F then prove that T satisfies a polynomial of degree n over F.

[7+7]

6. **a)** If V(F) is a finite dimensional vector space and $T \in A(V)$ is singular then prove that there exists $S \neq 0$ in A(V) such that TS=ST=0.

b) Let $S = \{\alpha_1, \alpha_2, \ldots, \alpha_m\}$ be an orthogonal set of nonzero vectors in an inner product space V(F). If β in V is in L(S) then prove that $\beta = \sum_{k=1}^{m} \frac{(\beta, \alpha_k)}{\|\alpha_k\|^2} \cdot \alpha_k$.

c) Prove that any orthogonal set of non zero vectors in an inner product space V(F) is linearly independent. **[5+5+4]**

7. **a)** Apply the Gram-Schmidt process to the basis {(1, 0, 1), (1, 0, -1), (0, 3, 4)} of $V_3(R)$ to obtain orthonormal basis with the standard inner product.

b) Prove that a linear transformation T on an inner product space Vis unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V. **[6+8]**

8. **a)** Let V(F) be an inner product space. If $T \in A(V)$ then prove that $T^* \in A(V)$ and

i) $(T^*)^* = T$

ii) $(S+T)^* = S^* + T^*$

iii) $(\lambda S)^* = \bar{\lambda} S^*$ (iv) $(TS)^* = S^* T^*$ for all $S, T \in A(V)$.

b) Let V(F) be an inner product space and $T \in A(V)$. Prove that T is self adjoint if and only if (Tx, x) is real for all $x \in V$. **[10+4]**

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KLE Society's

Raja Lakhamagouda Science Institute (Autonomous),

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First Semester M.Sc. Degree Examination Dec- 2018

AM03: GROUP THEORY

Duration: **3 Hrs** Max Marks: **70**

Instructions to candidates:

1) Attempt any Five Questions.
2) Figures to the right indicate full marks

1. **(a)** If G is a group and a$\in$ G then prove that O(a)=O(< a >).

(b) Prove that every subgroup of a cyclic group is cyclic.

(c) Define Euler's function. Find $\emptyset(7)$ and $\emptyset(12)$. **[5+5+4]**

2. **(a)** State and prove Lagrange's theorem for finite group.

(b) Define Normal subgroup. Prove that normalize of an element is a subgroup of a group.

(c) Define Factor group. If G is a group and G/Z(G) is a cyclic then prove that G is abelian group. **[5+5+4]**

3. **(a)** Define group homomorphism. Let $f:G \to G'$ be a group homomorphism from G into G' and if H is subgroup of G' then show that $f^{-1}(H)$ is subgroup of G.

(b) Let H and K be subgroups of G. If H is normal subgroup of G then prove that K/(H$\cap$K) is isomorphic to HK/H. **[6+8]**

4. **(a)** Define an equivalence class. If G is a group and a$\in$ G then prove that $|C_a|$=[G, N(a)], where C_a and N(a) are equivalence class and normalizer of a.

(b) If G is a finite group of order P^2 for some prime p then show that G is abelian group.

(c) Let G be a finite group of order n. If p is prime divisor of n then prove that G has an element of order p. **[5+4+5]**

5. **(a)** State and prove first Isomorphism theorem.

(b) Prove that number of p-sylow subgroups in G is equal to O(G)/O[N(p)], p is any p-sylow subgroup of G.

(c) Let H and K be subgroup of G such that GH x K then show that H and K are normal subgroups of G and G/K$\cong$H. **[5+5+4]**

6. **(a)** If H^*, H, K^*, K be subgroups of group G such that $H^* \trianglerighteq H$ and $K^* \trianglerighteq K$ then prove that $H^*(H \cap K)/H^*(H \cap K^*) \cong (H \cap K)/(H^* \cap K)(H \cap K^*)$

   (b) Define orbit of an element. Let a group G act on a set X. Prove that any two orbits are either equal or disjoint subsets of X and the union of all orbits equals X.

[8+6]

7. **(a)** State and prove Cayley theorem for a finite group.

   (b) Define symmetric group. Prove that for any non empty finite sets X and Y, $X \cong Y$ if and only if $S(X) \cong S(Y)$.

   (c) Define subnormal series of a group and give an example. **[5+6+3]**

8. **(a)** State and prove Jordan-Holder theorem for a finite group.

   (b) Define solvable group. Let N be a normal subgroup of a group G such that both N and G/N are solvable group. Show that G is solvable group. **[6+8]**

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KLE Society's

Raja Lakhamagouda Science Institute (Autonomous),

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First Semester M.Sc. Degree Examination Dec- 2018

AM04: REAL ANALYSIS

Duration: 3 Hrs **Max Marks: 70**

Instructions to candidates:

1) Attempt any Five Questions.
2) Figures to the right indicate full marks

1. a) State and prove Schroeder – Berstein theorem.

b) Show that the cardinal number $\aleph_0$ is the smallest transfinite cardinal number.

[10+4]

2. a) Show that the cardinal number of the power set A is 2^a where 'a' is the cardinal number of A.

b) Prove $\aleph_0 + \aleph_0 = \aleph_0$.

c) Show that the Hausdorff maximal principle implies Zorn's lemma.

[5+4+5]

3. a) Define metric space. Show that R^k is a metric space.

b) Show that every k-cell is compact in R^k.

c) Show that every neighbourhood of a point in a metric space is open set.

[4+5+5]

4. a) State and prove Heine-Borel theorem.

b) Show that every bounded infinite subset of R^k has a limit point in R^k.

c) Prove that every Cauchy sequence in R^k converges. **[6+3+5]**

5. a) Let (X, dx) and (Y, dy) be metric spaces and let $E \subset X$ and $p \in E$ such that p is a limit point of E. Let $f: E \to Y$ be a function. Show that f is continuous at p if and only if $\lim_{x \to p} f(x) = f(p)$.

b) State and prove Cauchy's Mean Value theorem.

c) Classify the discontinuous of a function. **[6+6+2]**

6. Let f: [a b] $\to$R be a bounded function and α:[a b] $\to$R be increasing function.

a) If P* is a refinement of a partition P of [a b], then show that (in usual notation) $L(P, f, \alpha) \leq L(P^*, f, \alpha)$.

b) Prove that f is Riemann-Stieltjes integrable over [a, b] w.r.t. α if and only if for every $\epsilon>0$, there exists a partition P of [a, b] such that $U(P, f, \alpha) - L(P,f, \alpha)<\epsilon$.

c) If f is continuous then prove that f is Riemann-Stieltjes integrable over [a, b] w.r.t.α.

[5+6+3]

7. a) Let $f\in R(\alpha)$ on [a, b] and let φ: [A, B] $\to$[a, b] be a strictly increasing continuous function such that φ^1 exists and $\varphi^1(x) > 0$. Define functions β and g by $\beta(y)=\alpha(\varphi(y))$ and $g(y)=f(\varphi(y))$.

Then show that $g\in R(\beta)$ on [A, B] and $\int_a^b f.d\alpha=\int_A^B g.d\beta$.

b) State and prove First Mean Value theorem for Riemann-Stieltjs integrable function on [a, b].

c) State and prove the first fundamental theorem of calculus. **[5+4+5]**

8. a) Define a function of bounded variation on [a b]. Show that every function of bounded variation on [a, b] is bounded and show that also the converse is not true.

b) State and prove Riemann rearrangement theorem. **[5+9]**

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KLE Society's

Raja Lakhamagouda Science Institute (Autonomous),

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First Semester M.Sc. Degree Examination Dec- 2018

AM05: ORDINARY DIFFERENTIAL EQUATION

Duration: **3 Hrs** Max Marks: **70**

Instructions to candidates:

1) Attempt any Five Questions.
2) Figures to the right indicate full marks

1. a) Let ϕ be any solution of $L(y) = y^{11} + a_1 y^1 + a_2 y = 0$ on an interval I containing a point x_0. Then prove that $\|\phi(x_0)\| \; e^{-k|x-x_0|} \le \|\phi(x)\| \le \|\phi(x_0)\| \; e^{k|x-x_0|}$ where $\forall x \in I$

$\|\phi(x)\| = \left[|\phi(x)|^2 + |\phi^1(x)|^2 \right]^{1/2}$ and $k = 1 + |a_1| + |a_2|$

b) Prove that two solutions $\emptyset_1$ and $\emptyset_2$ of L(y)=0 are linearly independent on an interval I if and only if $W(\emptyset_1, \emptyset_2)(x) \neq 0 \; \forall x \in I$. **[9+5]**

2. a) If ϕ_1, ϕ_2 are two solutions of L(y) = o on an interval I containing a point x_0, then prove that $W(\phi_1, \phi_2)(x) = e^{-a(x-x_0)} \; W(\phi_1, \phi_2)(x_0)$.

b) Using the method of variation of parameters solve $y^{11} + n^2 y = \sec nx \; \forall n \in N$.

[7+7]

3. a) Compute the solution of $y^{111} + y^{11} + y^1 + y = 1$ satisfying the condition , $\emptyset(0) = 0$, $\emptyset^1(0) = 1$ and $\emptyset^{11}(0) = 0$.

b) If $\alpha_1, \alpha_2, \alpha_3, \dots \alpha_n$ are any n constants and $x_0 \in R$ then prove that there exists a solution $\emptyset$ of L(y)=0 on R satisfying $\emptyset(x_0) = \alpha_1, \emptyset^1(x_0) = \alpha_2 \; \dots \; \emptyset^{n-1}(x_0) = \alpha_n$

[9+5]

4. a) Using Annihilator method solve $y^{11} + 4y = \cos x$.

b) Derive the Wronskian formula for n^{th} order homogeneous equation in the form

$W(\emptyset_1, \emptyset_2, \emptyset_3, \dots \emptyset_n)(x) = e^{-\int_{x_0}^{x} a_1(t).dt} \; W(\emptyset_1, \emptyset_2, \emptyset_3, \dots \emptyset_n)(x_0)$. **[7+7]**

5. a) Find the Eigen values and Eigen functions of $\frac{d}{dx}(xy^1) + \frac{\lambda}{x} y = 0$, $y^1(1) = y^1(e^{2\pi}) = 0$ where $\lambda > 0$.

b) Find the power series solution of $y^{11} - xy = 0$ about the ordinary point x=0. **[7+7]**

6. a) Derive the Rodrigue's formula for Legendre's equation in the form

$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2-1)^n$.

b) Find the general solution of $x^2 y^{11} - 5xy^1 - 9y = x3$. **[7+7]**

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7. **a)** Show that **(i)** $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$

(ii) $J_{-1/2}(x)=\sqrt{\frac{2}{\pi x}}\cos x$

b) Prove that **(i)** $T_{n+1}(x) + T_{n-1}(x) = 2xT_n(x)$

(ii) $nT_{n-1}(x) - nxT_n(x) = (1-x^2)\, T_n^1(x)$ **[6+8]**

8. **a)** Find the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3$ for $y^1 = 1+xy$, $y(0) = 1$.

b) Define Lipschitz condition. Show that the function $f(x,y) = x^2\cos^2 y + y\sin^2 x$ satisfies Lipschitz condition on $S = \{(x,y) \,/\, |x| \le 1, |y| < \infty\}$.

[7+7]

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KLE Society's

Raja Lakhamagouda Science Institute (Autonomous),

Belagavi.

First Semester M.Sc. Degree Examination December - 2018

AM07: COMPLEX ANALYSIS

Duration: 3 Hrs **Max Marks: 70**

Instructions to candidates:

1) Attempt any Five Questions.
2) All questions carry equal marks.
3) Figures to the right indicate full marks.

Q.1. (a) Prove that the function f(z) = xy+iy is continuous every where but not analytic.

(b) Define Laplace equation. If f(z) = u+iv is an analytic function then prove that u and v are harmonic function.

(c) If W=f(z) is a regular function of z then prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log|f'(z)| = 0.$

[4+5+5]

Q.2. (a) State and prove necessary condition for w = f(z) to be a conformal mapping.

(b) Evaluate the cross section of [2, 1-i, 1, 1+i].

(c) Prove that the power series and its derivative have the same radius of convergence.

[7+2+5]

Q.3. (a) If f: G → ℂ is analytic function and $\overline{B}$(a, r)⊆G then prove that

$$f^{n}(a) = \frac{n!}{2\pi i}\int_{\gamma}\frac{f(w)}{(w-a)^{n+1}}\,dw \text{ where } \gamma(t) = a+re^{it},\ 0 \le t \le 2\pi$$

(b) State and prove Cauchy's inequality.

(c) If f: ℂ → ℂ is an entire function with f(0)=1, f(1)=2 and f ′(0)=0. If ∃M>0 such that $|f''(z)| \le M$ ∀z∈ ℂ then find f(4). **[5+5+4]**

Q.4. (a) State and prove Fundamental theorem of algebra.

(b) If G is a connected open set and f: G→G is an analytic function then show that the following statements are equivalent.

(i) f=0 on G.

(ii) {z∈ G: f(z)=0}has a limit point in G.

(iii) There is a point a∈G such that $f^n(a)$=0 for each n≥0. **[5+9]**

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Q.5. (a) If **G** is an open subset of the plane and $f: G \to \mathbb{C}$ is an analytic function. If $\boldsymbol{\gamma}$ is a closed rectifiable curve in **G** such that $n(\gamma, w) = 0$ for all $w \in \mathbb{C} - G$. Then for $a \in G - \{\gamma\}$ prove that $f(a) = \frac{1}{2\pi i n(\gamma,a)} \int_{\gamma} \frac{f(z)}{z-a} dz$.

(b) If G be a region and $f: G \to G$ be a continuous function such that $\int f(z)dz = 0$ for every triangular path T in G. Then prove that f(z) is an analytic function in G.

[8+6]

Q.6. (a) State and prove Goursat's theorem.

(b) Find the Laurents series expansion of $f(z) = \frac{1}{z(z-1)(z-2)}$ in an annulus $0<|z|<1$.

i) $|z|<1$ and ii) $1 < |z| < 2$ **[10+4]**

Q.7. (a) State and prove Laurent's theorem.

(b) Evaluate $\int_0^{\pi} \frac{d\theta}{a+\cos\theta}$ **[9+5]**

Q.8.(a) State and prove Argument principle.

(b) With the help of argument principle, show that $\int_{|z|=\pi} \tan \pi z \, dz = -12i$.

(c) State and prove Rouche's theorem. **[6+3+5]**

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