K.L.E. Society's

Raja Lakhamagouda Science Institute

(Autonomous)

BELAGAVI.

SUBJECT

M.Sc - III Semester

Dec - 2018

(Mathematics)

QUESTION PAPER BOOKLET

Reg.No. | | | | | | | |

KLE Society's

Raja Lakhamagouda Science Institute (Autonomous),

Belagavi.

Third Semester M.Sc. Degree Examination Dec- 2018

CM01: FUNCTIONAL ANALYSIS

Duration: **3 Hrs** Max Marks: **70**

Instructions to candidates:

1) Attempt any Five Questions.
2) Figures to the right indicate full marks

1. a) Define norm on a vector space N. Show that a norm is a continuous mapping from N into R.

b) Show that the space ℓ^{∞} is normed linear space.

c) If N is a Banach space and if M is a closed subspace of N, then prove that the quotient space N/M is a Banach space. **[4+4+6]**

2. a) State and prove the Riesz lemma.

b) Let N and N^1 be normed linear spaces. Prove that a linear transformation $T:N\rightarrow N^1$ is continuous if and only if T is bounded.

c) Show that every convergent sequence in a normed linear space is a Cauchy sequence. **[6+6+2]**

3. a) State and prove the Hahn-Banach theorem and prove that for each non zero vector x_0 in a normed linear space N, there is a bounded linear functional f_0 on N such that $f_0(x_0)=\|x_0\|$

b) Let $1< p <\infty$. Prove that the dual space of ℓ^p is ℓ^q where $\frac{1}{p}+\frac{1}{q}=1$. **[8+6]**

4. a) State and prove open mapping theorem.

b) Show that a normed linear space N is a Banach space if and only if every absolutely convergent series in N is summable.

c) State merely the closed graph theorem. **[6+6+2]**

5. a) Suppose the dual space of a normed linear space is separable. Prove or disprove that the normed linear space is separable.

b) Let N^* and N^{**} be the first and second dual space of a normed linear space N. For each $x\in N$, defined F_x on N^* by $F_x(f)=f(x)$. Prove that

(i) F_x is linear continuous mapping from N^* into N^{**} and $\|F_x\| = \|x\|$

(ii) The mapping $x\rightarrow F_x : N\rightarrow N^{**}$ is linear mapping. **[7+7]**

6. **a)** State and prove the Uniform boundedness principle.

b) Let x and y be a Hilbert space H. Prove that

(i) $|< x, y >| \leq \|x\|. \|y\|$

(ii) $2(\|x\|)^2 + \|y\|^2) = \|x + y\|^2 + \|x - y\|^2$ **[7+7]**

7. **a)** Show that every closed convex subset of a Hilbert space contains a unique vector of smallest norm.

b) Let S be a non empty subset of a Hilbert space H. Show that its orthogonal complement is a closed linear subspace of H.

c) Let $M^{\perp}$ be the orthogonal complement of a closed subspace M of a Hilbert space H. Show that H = M$\oplus M^{\perp}$. **[5+4+5]**

8. **a)** Let $\{e_1, e_2, \ldots e_n\}$ be an orthogonal set in a Hilbert space H. Prove that for each x$\in$H, $\sum_{i=1}^{n}|< x, e_i|^2 \leq \|x\|^2$.

b) Let $\{e_i\}$ be a complete orthogonal set in a Hilbert space. Prove $x \perp e_i$ for all i implies x=0.

c) Let T be an operator on Hilbert space H. Show that there is a unique operator T^* on H such that <Tx, y> = <x, T^*y> for all x, y$\in$H. Also prove that $\|T^*\| = \|T\|$ **[3+3+8]**

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Third Semester M.Sc. Degree Examination Dec- 2018

CM02: DIFFERENTIAL GEOMETRY

Duration: **3 Hrs** Max Marks: **70**

Instructions to candidates:

1) Attempt any Five Questions.
2) Figures to the right indicate full marks

1. a) Define directional derivative of a real valued function f in the direction of $\overline{v}$ at a point P. If $\overline{v_p}=(v_1, v_2, v_3)$ is a tangent vector at p and f is a real valued function on R^3, then prove that $\overline{v_p}[f] = \sum_{i=1}^{3} v_i \frac{\partial f}{\partial x_i}(p)$

b) If $\overline{V} = x^2y\overline{U}_1 + y^2z\overline{U}_2 + z^2x\overline{U_3}$ and f is a real valued function on R^3 given by $f=10x^2+y^2+8z^2$ find $\overline{V}[f]$.

c) Prove that a regular curve α can be re-parameterized to a unit speed curve β.

[5+4+5]

2. a) If β is a unit speed curve then prove that $\overline{T'}=K\overline{N}$, $\overline{N'} = -K\overline{T} + \tau\overline{B}$ and $\overline{B'} = -\tau\overline{N}$.

b) Compute the Frenet apparatus for a unit speed curve $\beta(s) = (a\cos(s/c), a\sin(s/c), bs/c)$ where $c^2 = a^2 + b^2$ **[7+7]**

3. a) Derive the Frenet approximation of a unit speed curve near the origin.

b) Let β be a unit speed curve. Prove that β is a plane curve if and only if its torsion τ is zero. **[7+7]**

4. a) Compute the Frenet apparatus for an arbitrary speed curve $\alpha(t) = (t\cos t, t\sin t, t)$ at t=0.

b) Define a covariant derivative of a vector field $\overline{W}$ w.r.t. a vector field $\overline{V}$. If $\overline{W} = \sum_{i=1}^{3} w_i\overline{U_i}$ is a vector field on R^3 and $\overline{v}$ is a tangent vector at p then prove that $\nabla_{\overline{v}}\overline{W} = \sum_{i=1}^{3} \overline{v_p}[w_i]\overline{U_i}(p)$ **[7+7]**

5. a) Let g be a differentiable real valued function on R^3 and c be any constant. Prove that the subset M of R^3 given by M: g(x, y, z) = c is a surface if and only if dg≠0 at any point of M.

b) Show that the figure of eight is not a surface. **[8+6]**

6. **a)** Let M be a surface. For each point p$\in$ M show that the shape operator
$S_p : T_p(M) \rightarrow T_p(M)$ is a linear operator on $T_p(M)$.

b) For a saddle surface M: z=xy show that S $(a\overline{u_1}+b\overline{u_2}) = a\overline{u_2} + b\overline{u_1}$ where S is a shape operator and $\overline{u_1} = \overline{U_1}(0)$ and $\overline{u_2} = \overline{U_2}(0)$. **[7+7]**

7. **a)** If $\bar{v}$ and $\bar{w}$ are linearly independent tangent vectors at a point p of a surface M then prove $S(\bar{v}) \times S(\bar{w}) = k(p)\, \bar{v} \times \bar{w}$ and $S(\bar{v}) \times \bar{w} + \bar{V} \times S(\bar{w}) = 2H(p)\, \bar{v} \times \bar{w}$ where S is a shape operator on M and K(p) and H(p) are Gaussian curvature and Mean Curvature M at the point p.

b) Let X be a patch in a surface M. With usual meaning of symbols prove that the Gaussian curvature K and the mean curvature H are given by $K = \frac{l.n-m^2}{EG-F^2}$

$H = \frac{G.l+E.n-2Fm}{2(EG-F^2)}$ **[7+7]**

8. **a)** Find the centre and radius of a circle given by $\beta(s) = (\frac{4}{5}\cos s, 1-\sin s, -\frac{3}{5}\cos s)$

b) Obtain normal and tangent vector fields on a surface of a sphere
$\Sigma: (x^2 + y^2 + z^2) = r^2$.

c) Let S be a shape operator on a surface M. Show that characteristic polynomial of S is given by $P(k) = k^2 - 2Hk + K$. **[5+5+4]**

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Third Semester M.Sc. Degree Examination Dec- 2018

AM04: LATTICE THEORY

Duration: 3 Hrs **Max Marks: 70**

Instructions to candidates:

1) Attempt any Five Questions.
2) Figures to the right indicate marks

Important Note: 1. On completing answers, compulsorily draw diagonal lines on the remaining blank pages. 2. Any revealing of identification, appeal to valuator and / or equations written will be treated as malpractice.

1. **a)** Define a modular lattice. Prove that the lattice of all normal subgroups of a group is a modular lattice.

b) Prove that a lattice L is modular if and only if whenever $a \geq b$ and $a \wedge c = b \wedge c$, $a \vee c = b \vee c$ for some $c \in L$ then $a=b$. **[7+7]**

2. **a)** Prove that a distributive lattice is modular. Is the converse true? Justify your answer.

b) Prove that in a bounded distributive lattice an element can have only one complement.

c) If L is not distributive is it totally ordered? **[6+6+2]**

3. **a)** Prove that two bounded lattices A and B are complement iff AxB is complemented.

b) Give an example to show that union of two ideals need not be an ideal.

c) Prove that union of two ideals in a Lattice L is an ideal if and only if one of them is contained in the other. **[6+2+6]**

4. **a)** Prove that the set of all principal ideals P of a lattice L forms a lattice under $\subseteq$ and is a sublattice of the ideal lattice I(L) of L. Also show that this lattice P is isomorphic with L.

b) Prove that a lattice L is a chain if and only if all the ideals in L are prime.

c) Let I be a prime ideal of lattice L. Show that L-I is a dual prime ideal.

[6+5+3]

5. **a)** Prove that two lattices L and M are modular if and only if LxM is modular.

b) If a and b are elements of a modular lattice L then prove that $[a \wedge b, a] \cong [b, a \vee b]$.

c) Show that every chain is a distributive lattice. **[6+5+3]**

6. **a)** Prove that a modular lattice is non distributive if and only if it contains a sublattice isomorphic with M_5.

b) Prove that a lattice L is distributive if and only if $a \vee c = b \vee c$ imply $a=b$, where $a, b \in L$. **[8+6]**

7. **a)** Define a congruence relation on a lattice L.

Let L be a stone lattice. Define ~ in L by $a \sim b \Leftrightarrow a^* = b^*$. Prove that ~ is a congruence relation.

b) Prove that any quotient lattice of a lattice L is a homomorphic image of L.

c) If (H) is a congruence relation on a lattice L, show that $a \equiv b(H) \Leftrightarrow a \wedge b \equiv a \vee b(H)$

[4+4+6]

8. **a)** State only Stones theorem. Let L be a distributive lattice and $a, b \in L$ such that $a \neq b$. Prove that $\exists$ a prime ideal P containing exactly one of them.

b) Let L be a bounded distributive lattice with $0 \neq 1$. Then prove that L is a Boolean lattice iff P(L), the set of prime ideals of L is an ordered. **[6+8]**

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KLE Society's

Raja Lakhamagouda Science Institute (Autonomous),

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Third Semester M.Sc. Degree Examination Dec- 2018

CM05: DISTRIBUTION THEORY

Duration: **3 Hrs** Max Marks: 70

Instructions to candidates:

1) Attempt any Five Questions.
2) Figures to the right indicate full marks

1. a) Define the spaces D(R) and $C_c(R)$. Show that D(R) is dense in $C_c(R)$.

b) Let fn(t)=sin nt, n=1,2,___. Show that fn→ 0 in $D'(R)$ but {fn} diverges as sequence of functions. **[8+6]**

2. a) Let f∈ $D'(R)$ and $\varphi \in C^{\infty}(R)$. Define φf in $D'(R)$ and derives the Leibnitz rule for computing $(\varphi f)^{(n)}$.

b) For f∈ $D'(R)$, prove $\frac{f(t+h)-f(t)}{h} \to f'$ in $D'(R)$.

c) Show that $\delta: \varphi \to \varphi(0)$ is a distributive in$D'(R)$. **[6+6+2]**

3. a) State and derive the local boundedness property of distribution in $D'(R)$.

b) Define $P_v \frac{1}{x}$ in $D'(R)$. Show that the derivative of $P_v \frac{1}{x}$ is $\log|x|$. **[7+7]**

4. a) Define a primitive $f^{(-1)}$ of f∈ $D'(R)$. Show that $f^{(-1)} \in D'(R)$.

b) Define the space S(R). Show that $\varphi \in C^{\infty}(R)$ is in S(R) if and only if $\lim_{t\to\infty} |t^m| \, |\varphi^{(k)}(t)| = 0$ for m, k=0,1,2,3,___.

c) Show that the space D(R) is dense in S(R). **[5+4+5]**

5. a) State and prove the boundedness property of distributives of slow growth.

b) Give an example of a distribution f of slow growth and a C^{∞}-function φ such that $\varphi f \notin S'(R)$.

c) Let $\varphi(t) = e^{-t^2}$. Prove that $e^{-t^2} \notin D(R)$ but $e^{-t^2} \in S(R)$. **[6+5+3]**

6. a) Define f(t)∈D'_t and $g(\tau) \in D'_\tau$.Show that their direct product f(t)xg(τ) is in$D'_{t,\tau}$.

b) Define convolution f∗g of f∈ $D'(R)$ and g∈ $D'(R)$. Obtain

(i) $\delta * f$ **(ii)** $\delta^{(m)} * f$. **[8+6]**

7. **a)** Define the Fourier transform of a testing function of rapid descent and show that it is again a testing function of rapid descent.

b) Define the Fourier transform of f$\in S'(R)$ and compute the Fourier transform of

(i) $f(t)=t^n$ **(ii)** $f(t)=e^{i\alpha t}$, $\alpha \in R$. **[7+7]**

8. **a)** Define the spaces D_1 and DoI. Show that D_1 is dense in DoI.

b) Compute $t^n \delta^{(n)}(t)$ in $D'(R)$.

c) Let f$\in D'(R)$ and $\tau \in R$. Define $f(t-\tau)$ and show that $f(t-\tau)$ is in $D'(R)$. **[7+3+4]**

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2.6 OPEN ELECTIVE COURSE (MATHEMATICAL STATISTICS –I) (60 Hours)

Unit 1: Basic Statistics: Meaning of statistics as numerical information and as a Science of data analysis. Measurement scales: Nominal, Ordinal, Interval and ratio. Presentation of data: Classifications of data, Variable: Discrete and Continuous variables , Frequency distribution.

Measures of location, Definition of A.M, G.M, H.M and their Properties Theoretical problems, Partition values.

Measure of dispersion, Mean deviation, Standard deviation, Variance, Properties of S.D., Coefficient of variation. Definition of Moments, Skewness.

Unit 2: Probability:

Introduction to Probability – Basic concepts, Mathematical and Axiomatic probability Axioms of probability, Addition theorem, conditional probability, independent events, Multiplication rule, Baye's Theorem- computation of probabilities.**Random variable** :Definition of Random variables - Discrete and Continuous random variable. Probability mass function (pmf) and Probability density function (pdf) and simple examples. Expectation, properties of expectation, Moment generating function (m.g.f) and its properties. Variance, Bivariate distribution function: Joint, Marginal, Conditional distributions for discrete and continuous variates, Addition and Multiplication law of Expectation. Covariance and correlation by expectation.

Unit 3: Standard Discrete and Continuous distribution

Discrete distributions – Bernoulli distribution – definition, Binomial distribution and Poisson distribution , definition, examples, MGF, moments, Additive property of B.D. and P.D. Fitting of Binomial and Poisson to the given data.

Normal distribution, definition, examples, normal curve – properties , MGF, moments, Standard Normal Vitiate and its properties.

Unit: 4 Correlation and Regression:

Definition, Types of Correlation, Karl Pearson's Correlation Coefficient and its Properties. Definition and derivation of Rank correlation coefficient and properties. Regression definition and meaning ,Regression coefficients, Regression lines and their properties.

Unit 5: Test of Significance:

Tests of significance, Hypothesis, Simple hypothesis, Composite hypothesis, Null hypothesis, Alternative hypothesis, Type-I & Type-II errors, critical region and level of significance, one tailed and two tailed test, critical values, procedure of testing of hypothesis, P-values. **Large Sample Tests: Test** of significance for large samples -single mean and difference of means.

Small sampling Tests: Application of chi-square distribution, t-distribution and F-distribution

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Tests of significance, Hypothesis, Simple hypothesis, Composite hypothesis, Null hypothesis, Alternative hypothesis, Type-I & Type-II errors, critical region and level of significance, one tailed and two tailed test, critical values, procedure of testing of hypothesis, P-values. **Large Sample Tests: Test** of significance for large samples -single mean and difference of means.

Small sampling Tests: Application of chi-square distribution, t-distribution and F-distribution

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Third Semester M.Sc. Degree Examination Dec - 2018

CM08: MATHEMATICAL STATISTICS

Duration: **3 Hrs** Max Marks: **70**

Instructions to candidates:

1) Attempt any Five Questions.
2) Scientific calculator permiitted
3) Statistical table will be supplied on request.

1. a) Explain Mathematical approach of probability and prove that $0 \leq P(\text{event}) \leq 1$.

b) Define the term conditional probability and state Multiplication theorem for three dependent events.

c) A factory produces a certain type of outputs by three types of Machines as follows. Machine I: 3,000 units Machine II: 2500 units and Machine III: 4500 units. The fractions of defectives for the machine are 1%, 1.2% and 2% respectively. An item is drawn at random from the day's production run and is found to be defective. What is the probability that it comes from Machine I, Machine II and Machine III.

[4+4+6]

2. a) What is the probability of getting 53 Sundays in a (i) leap year (ii) Non-leap year.

b) Explain Binomial distribution.

c) Obtain Mean and Variance of Binomial distribution. **[4+5+5]**

3. a) The incidence of an occupational disease in an industry is such that the workers have 25% chance of suffering from it. What is the probability that out of 5 workers, at the most two contract the disease?

b) Obtain recurrence relation between successive terms of Poisson distribution.

c) A box contains 200 fuses, out of which 2% are defective. What is the probability that a box contains **(i)** defective fuses **(ii)** 3 or more defective fuses.

[4+4+6]

4. a) In a normal distribution, 15% items are below 35 and 10% items are above 65. Find the mean and standard deviation.

b) Distinguish between correlation and regression.

c) What is the scatter diagram? How do you interpret scatter diagram. **[5+4+5]**

5. a) Calculate the co-efficient of correlation between the weights of mothers and babies

Mother(kgs)	54	60	49	57	68	55	62	60	58
Baby(kgs)	3.4	3.2	3.6	4.0	2.4	3.6	4.1	2.5	3.5

b) State the properties of regression coefficient.

c) 6 students are ranked in sports and studies as follows. Find the co-efficient of rank correlation.

Students	1	2	3	4	5	6
Rank in sports	1	2	3	4	5	6
Rank in studies	4	5	6	2	1	3

[6+4+4]

6. a) The following are the heights of 10 persons and one each of their sons. Obtain two regression equations and estimate the most probable heights of a person whose father is 184cm.

father (cms)	158	160	163	165	167	170	167	172	177	181
sons(cms)	163	158	167	170	160	180	170	175	172	175

b) Explain Null hypothesis and alternative hypothesis. [10+4]

7. a) Write the procedure to test: Test for population mean

b) A random sample of 400 tins of vanaspati has mean weight 4.96kg and standard deviation 0.4kg. Test at 1% level of significance that the average weight of tins of vanaspati is 5kg.

c) Mention the conditions for the validity of Chi-square test. [5+5+4]

8. a) Mention the applications of t-distribution.

b) In sample of 8 observation the sum of the square of deviations of sample values from sample mean was 84.4 and in the other sample of 10 observation it was 102.6. Test whether this difference is significant at 5% level with degrees of freedom for $n_1=7$ and $n_2=9$ is 3.29.

c) List advantages and disadvantages of Randomized block design. [4+6+4]

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Third Semester M.Sc. Degree Examination Dec- 2018

CM09: FUZZY MATHEMATICS

Duration: 3 Hrs Max Marks: 70

Instructions to candidates:

1) Attempt any Five Questions.
2) Figures to the right indicate full marks.

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1. a) Let $f : X \to Y$ be a function , then prove that

i) $A \leq f^{-1}[f(A)]$ for every fuzzy subset A of X.

ii) $f(A \cup B) = f(A) \cup f(B)$ for any fuzzy subset A, B of X

b) Define with an example

i) α-cut of fuzzy set

ii) support of a fuzzy set.

c) Let X be a set. Let A and B be any two fuzzy subsets of X. Let $\alpha, \beta \in [0, 1]$ then prove that **(i)** $\alpha_{A\cap B} = \alpha_A \cap \alpha_B$ **(ii)** $A \subseteq B \leftrightarrow \alpha +_A \leq \alpha +_B$

[6+4+4]

2. a) Prove that a fuzzy set A on $\mathbb{R}$ is convex if and only if $A[\lambda x_1 + (1-\lambda)x_2] \geq \min\{A(x_1), A(x_2)\}$.

b) Let $A = \frac{0.2}{x_1} + \frac{0.4}{x_2} + \frac{0.6}{x_3} + \frac{0.8}{x_4} + \frac{1}{x_5}$ and $B = \frac{0.4}{x_1} + \frac{0.2}{x_2} + \frac{0.5}{x_3} + \frac{0.9}{x_4} + \frac{0.1}{x_5}$.

Find **(i)** $A \cup \overline{A}$ **(ii)** S(A, B)

c) Let A be a fuzzy subset of X and let $\alpha, \beta \in [0,1]$ then show that

$${}^{\alpha}A = \bigcap_{\beta<\alpha} {}^{\beta-}A = \bigcap_{\beta<\alpha} {}^{\beta+}A$$

[5+4+5]

3. a) State and prove Second decomposition theorem .

b) Define Zadeh's Extension Principle. Let $f: X \to Y$ be a function. Then for any $A \in F(x)$ and $\forall\, \alpha \in [0, 1]$ prove that $f(\alpha_A) \subset \alpha_{[f(A)]}$. Is the converse true. Justify?

c) If $A(x) = \frac{x}{x+2}$ for all $x \in [0, 10]$ then find f(A). **[6+6+2]**

4. a) Show that the Yager's class of fuzzy complement is involutive.

b) Define equilibrium point of a fuzzy complement with an example. Prove that every fuzzy complement have atmost one equilibrium point.

c) If C is continuous fuzzy complement then prove that C has an unique equilibrium point.

[5+5+4]

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5. **a)** Define dual point of a point a $\in[0, 1]$ with an example. Let C be fuzzy complement with equilibrium point e_c. Then show that $d_{e_c} = e_c$.

b) Define t-norm. Prove that standard fuzzy intersection is the only t-norm which is idempotent. **[7+7]**

6. **a)** Prove that $\max(a, b) \le u_w(a,b) \le u_{max}(a,b)$ for all a, b$\in [0, 1]$, where $u_{max}(a,b)$ is Yager's class of t-conorm and u_{max} is drastic union of t-conorm.

b) Prove that $i_{min}(a, b) \le i(a,b) \le \min(a,b)$ for all a, b$\in [0, 1]$, where i_{min} is drastic intersection of t-norm and i is any other t-norm.

c) State Demorgan's law for fuzzy sets. **[6+6+2]**

7. **a)** Show that $<$ max (0, a+b-1), min (1, a+b), c $>$ is a dual triple where c is a standard fuzzy complement.

b) Show that $<$i, u, c$>$ is a dual triple, where u is t-conorm, c is an involute fuzzy complement and i(a, b) = c $<$ u(c(a), c(b)) $>$ for all a, b$\in$ [0, 1] is a t-norm.

[7+7]

8. **a)** Let A$\in$ F(R). Then prove that A is a fuzzy number if and only if there exists a closed interval [a, b] $\neq \emptyset$ such that A(x) = 1 if x$\in$ [a, b]

$$A(x) = \ell(x) \text{ if } x \in (-\infty, a)$$
$$= r(x) \text{ if } x \in (b, \infty)$$

Where $\ell: (-\infty, a) \to [0, 1]$ is monotonic increasing and continuous from the right such that $\ell(x)=0$ for $x \in (-\infty, w_1)$ and $r : (b, \infty) \to [0,1]$ is monotonic decreasing and continuous from the left such that $r(x)=0$ for $x \in (w_2, \infty)$.

b) Let R be the set of all fuzzy numbers and let MIN and MAX be the binary operation on R. Then prove that

(i) MIN [A, MIN (B, C)] = MIN [MIN (A, B), C]

(ii) MAX [A, MIN (B, C)] = MIN [MAX (A, B), MAX(A, C)]. **[7+7]**

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