K.L.E. Society's

Raja Lakhamagouda Science Institute

(Autonomous)

BELAGAVI.

SUBJECT

M.Sc. Mathematics II Semester

June - 2015

QUESTION PAPER BOOKLET

Reg.No. | | | | | | | | |

KLE Society's

Raja Lakhamagouda Science Institute (Autonomous)

Belagavi.

Second Semester M.Sc. Degree Examination May/June - 2015

BM01: GENERAL TOPOLOGY

Duration: **3 Hrs** Max Marks: **70**

Instructions to candidates:

1) Attempt any Five Questions.
2) All questions carry equal marks.
3) Figures to the right indicate full marks.

Q1. a) Let (X,τ) be a topological space. Prove that a subcollection $\mathfrak{B}$ of τ is a base for τ if and only if every τ-open set can be expressed as the union of members of $\mathfrak{B}$.

b) Let (X,τ) be a topological space and $A \subseteq X$. Then prove that A is a τ- open set if and only if A contains a τ– neighbourhood of each of its points .

c) Prove that the continuous image of a connected set is connected. **[5+5+4]**

Q2. a) Prove that a topological space (X,τ) is compact if and only if every collection of closed subsets of X with the finite intersection property has a non – empty intersection.

b) Let (X,τ) be a topological space and let $\{ \mathbf{C}_\lambda / \lambda \in \Delta\}$ be a non – empty collection of connected subsets of X such that $\cap\{ \mathbf{C}_\lambda / \lambda \in \Delta\} \neq \phi$. Then that $\cup\{ \mathbf{C}_\lambda / \lambda \in \Delta\}$ is connected set.

c) Let (X,τ) be a topological space and E be a connected subset of X. Then prove that $\overline{E}$ is connected. **[5+5+4]**

Q3. a) Prove that every compact topological space has the Bolzano – Weirstrass property.

b) Give an example of a connected Hausdroff space which is not compact.

c) Prove that every closed subspace of a Lindelof space is a Lindelof space.

[6+3+5]

 PTO

Q4. a) Let X be a Hausdroff topological space and let A, B be disjoint compact subsets of X. Prove that there exist disjoint open sets G and H such that $A \subseteq G$ and $B \subseteq H$.

b) Prove that every convergent sequence in a Hausdroff space has a unique limit.

c) Give an example to show that a T_3 – space need not be a T_4 – space.

[5+5+4]

Q5. a) Prove that every regular Lindelof space is normal.

b) Prove that every compact Hausdroff space is regular. **[8+6]**

Q6. a) Prove that a topological space (X, τ) is normal if and only if for any closed set F and an open set G containing F, there exists an open set V such that $F \subseteq V$ and $\overline{V} \subseteq G$.

b) Prove that a closed subspace of a normal space is normal .

c) Prove that a normal space is completely regular if and only if it is regular.

[6+4+4]

Q7. a) Define the product space X x Y of two topological spaces X and Y. Prove that the projections π_x and π_y are continuous and open mappings.

b) Let X and Y be two topological spaces. Prove that the product space X x Y is compact if and only if X and Y are compact.. **[7+7]**

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Second Semester M.Sc. Degree Examination May/June - 2015

BM02: RINGS AND FIELDS

Duration: **3 Hrs** Max Marks: **70**

Instructions to candidates:

1) Attempt any Five Questions.
2) All questions carry equal marks.
3) Figures to the right indicate full marks.

Q1. a) State and prove First isomorphism theorem.

b) Prove that every integrable domain can be embedded in a ring. **[7+7]**

Q2. a) Prove that a finite integrable domain is a field.

b) Find the units of Gaussian integer and show that they form a multiplicative abelian group.

c) If U is an ideal of the ring R, then prove that $\frac{R}{U}$ is a ring **[5+5+4]**

Q3. a) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is a field.

b) Prove that a Euclidean ring possesses a unit element.

c) Let R be an integral domain with unit element and suppose that for a, b $\in$ R both a|b and b|a are true. Then prove that a = ub, where u is a unit in R.

[5+5+4]

Q4. a) If p is a prime element in the Euclidean ring R and $p|a_1, a_2, \ldots a_n$ then prove that p divides atleast one $a_1, a_2, \ldots\ldots a_n$

b) Prove that F[x] is an integral domain.

c) Prove that any polynomial in F[x] can be written in a unique manner as a product of irreducible polynomials in F[x]. **[5+4+5]**

Q5. a) Prove that product of two primitive polynomials is primitive.

b) State and prove the Einstein's criterion. **[5+9]**

 PTO

Q6. a) If R is a unique factorization domain, then prove that R[x] is so.

b) If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F. Moreover [L:F] =[L:K][K:F]. **[5+9]**

Q7. a) Prove that the element $a \in K$ is algebraic over F if and only if F(a) is a finite extension of F.

b) Construct splitting Field over Q for the polynomial $x^3 - 1$. Also find the degree of extension over Q. **[7+7]**

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KLE Society's

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Second Semester M.Sc. Degree Examination May/June - 2015

BM03: ADVANCED CALCULUS

Duration: 3 Hrs — Max Marks: 70

Instructions to candidates:

1) Attempt any Five Questions.
2) All questions carry equal marks.
3) Figures to the right indicate full marks.

Q1. a) Suppose $\{f_n\}$ is a sequence of differentiable functions on $[a,b]$ such that $\{f_n(x_0)\}$ converges for some point $x_0 \in [a,b]$. If $\{f_n^1\}$ converges uniformly on $[a,b]$ then show that $\{f_n\}$ converges uniformly on $[a,b]$ and

$$\frac{d}{dx}\lim_{n\to\infty} f_n(x) = \lim_{n\to\infty}\frac{d}{dx} f_n(x),\ a \le x \le b.$$

b) Show that the sequence $\{f_n\}$ where $f_n(x) = nx\,e^{-nx^2}$, $x \ge 0$ does not converge uniformly on $[a,b]$ where $b > 0$

c) State and prove Cauchy criteria for uniform convergence. **[7+3+4]**

Q2. a) Let $\{f_n\}$ be a sequence of Riemann – Stieltjes integrable functions on [a,b] with respect to a real – valued increasing function α on $[a,b]$. If $f_n \to f$ uniformly on $[a,b]$, then show that f is Riemann – Stieltjes integrable function on $[a,b]$ and $\lim_{n\to\infty}\int_a^b f_n(t)d\alpha t)d = \int_a^b f(t)d\alpha(t)$.

b) State and prove Dirichlit's test for uniform convergence. **[8+6]**

Q3. a) Define the mean – convergence for a sequence of Riemann integrable functions on $[a,b]$. Prove that the uniform convergence of a sequence implies the mean convergence. Is the converse true?. Justify your answer.

b) Define Directional derivative of a vector field . If $T \in L(R^n, R^m)$ then compute all directional derivatives of T. **[9+5]**

Q4. a) If $T \in L(R^n, R^m)$, then show that $\|T\| < \infty$ and T is uniformly continuous.

b) Let f be a vector field from an open subset E of $\mathbf{R^n}$ into $\mathbf{R^m}$. Define the total derivative $\mathbf{f^1}(x)$ of f at $x \in \mathbf{E}$ and show that it is unique.

c) Compute the total derivative of a constant vector field . **[5+6+3]**

Q5. a) State and prove chain rule.

b) Let $\mathbf{f}: \mathbf{E} \to \mathbf{R^m}$ be differentiable at $x \in$ E. Show that the partial derivatives $\mathbf{D_i f_j}(x)$ exist and $\mathbf{f^1}(x)\mathbf{e_j} = \sum_{i=1}^{m} \mathbf{D_i f_j}(x)\mathbf{y_i}$ where $\{\mathbf{e_1, e_2, \ldots .. e_n}\}$ and $\{\mathbf{y_1, y_2, \ldots \ldots y_m}\}$ are the standard bases for R^n and R^m respectively and $\mathbf{f} = (\mathbf{f_1, f_2, \ldots . f_m})$. **[7+7]**

Q6. a) State and prove the contraction principle.

b) State and prove linear version of the implicit function theorem.

c) If f is differentiable vector field at x, then show it is continuous at x. **[6+5+3]**

Q7. a) State and prove the mean value theorem for the vector field.

b) Use the method of Lagrange multipliers to determine all critical points of a scalar field $\mathbf{f(x,y,z) = x + y + 2z}$ on the surface $\mathbf{x^2 + y^2 + z^2 = 3}$.

c) Find the scalar field from R^2 into R whose gradient is given by the vector field defined by $(\mathbf{3 + 2xy, x^2 - 2y^2})$. **[6+4+4]**

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KLE Society's

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Second Semester M.Sc. Degree Examination May/June - 2015

BM04: MEASURE AND INTEGRATION

Duration: **3 Hrs** Max Marks: **70**

Instructions to candidates:

1) Attempt any Five Questions.
2) All questions carry equal marks.
3) Figures to the right indicate full marks.

Q1. a) Define outer measure $\mathbf{m}^*$ on $\mathbf{P(R)}$. Show that $\mathbf{m^*(I)} = \ell\,\mathbf{(I)}$ for an interval I.

b) Define a measurable set and show that the union of two measurable sets is a measurable set.

c) Prove that the cantor set has measure zero. **[6+5+3]**

Q2. a) Let $\{\mathbf{E}_n\}_{n=1}^{\infty}$ be a sequence of measurable sets such that $\mathbf{E_{n+1}} \subseteq \mathbf{E_n}$ for each n.

If $\mathbf{m(E_1)} < \infty$ then show that $m\left(\bigcap_{n=1}^{\infty} \mathbf{E_n}\right) = \lim_{n\to\infty} \mathbf{m(E_n)}$.

b) If E is a measurable set then prove that there is a $\mathbf{G}_\delta$ - set **G** such that $\mathbf{E} \subseteq \mathbf{G}$ and $\mathbf{m^*(G - E) = 0}$.

c) Prove that every subset of R with positive outer measure has a non – measurable subset. **[5+4+5]**

Q3. a) Show that a function f defined on a measurable set is measurable if and only if $\mathbf{f^{-1}(G)}$ is measurable for each open set **G** in **R.**

b) Prove or disprove that the product of two measurable functions is measurable.

c) Show that a continuous function on a measurable set is measurable. **[6+4+4]**

Q4. a) State and prove Egoroff's theorem.

b) If φ and ψ are simple functions on **E** which vanish outside a set of finite measure then prove that $\int_E a\varphi + b\psi = a\int_E \varphi + b\int_E \psi$ for all real numbers a and b

[7+7]

Q5. a) State and prove the bounded convergence theorem

b) If $\{\mathbf{f_n}\}_{n=1}^{\infty}$ is a sequence of non – negative measurable functions on E such that $\mathbf{f_n \to f}$ a.e on **E**, then prove that $\int_E \mathbf{f} \le \liminf_{n\to\infty} \int_E \mathbf{f_n}$.

c) Let $\mathbf{f(x)} = \begin{cases} \mathbf{x}^{-1/3} & \text{if } \mathbf{0 < x \le 1} \\ \mathbf{0} & \textbf{if } \mathbf{x = 0} \end{cases}$

Compute $\int_0^1 \mathbf{f}$ **[5+5+4]**

Q6. a) If f and g are integrable functions on **E,** then show that **f +g** is integrable on **E** and $\int_E \mathbf{f+g} = \int_E \mathbf{f} + \int_E \mathbf{g}$.

b) State and prove Lebesgue dominated convergence theorem.

c) If f is integrable on **E,** then show that f is finite **a.e**. on **E.** **[5+5+4]**

Q7. a) If f is an integrable function on **[a,b]** and if $\mathbf{F}(x) = \int_a^x \mathbf{f(t)\ dt + c}$, where c is a constant then show that F is continuous function of bounded variations on **[a,b].**

b) State and prove Minkowoski's inequality for $\mathbf{L^p(E)}$ where $\mathbf{1 \le p < \infty}$.

[7+7]

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Second Semester M.Sc. Degree Examination May/June - 2015

BM05: COMPLEX ANALYSIS

Duration: **3 Hrs** Max Marks: **70**

Instructions to candidates:

1) Attempt any Five Questions.
2) All questions carry equal marks.
3) Figures to the right indicate full marks.

Q1. a) Derive sufficient condition for the function **f(z)** to be analytic.

b) Prove that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f^1(z)|^2$.

c) Show that the function $u = e^x(x\cos y - y\sin y)$ is harmonic. Find its harmonic conjugate and corresponding function. **[6+4+4]**

Q2. a) Prove that every power series represents an analytic function at all points within the circle of convergence and its derivatives can be obtained by term wise differentiation of a given power series.

b) Find the radius of convergence of the power series $\sum \frac{n!}{n^n}$.

c) Evaluate $\int_{-2+i}^{5+3i} z^3\, dz$. **[6+3+5]**

Q3.a) State and prove Cauchy's theorem for triangle

b) State and prove extension of Cauchy's theorem. **[9+5]**

Q4.a) State and prove fundamental theorem of Algebra.

b) State and prove Morera's theorem.

c) Evaluate $\int \frac{\cos\pi(Z^2)}{(z-1)(z-2)}\, dz$. c: $|z| = 3$. **[5+5+4]**

Important Note: 1. On completing answers, compulsorily draw diagonal lines on the remaining blank pages. 2. Any revealing of identification, appeal to valuator and / or equations written will be treated as malpractice.

Q5. a) State and prove Laurent's Expansion.

b) Expand $\frac{1}{(Z+2)(1+z^2)}$ when $1 < |z| < 2$ **and** $|z| > 2$. **[8+6]**

Q6. a) Define isolated essential singularity. If function $f(z)$ is analytic in $D_R(\alpha)$ then f has a zero of order k at the point $z = \alpha$ if and only if f can be expressed in the form $f(z) = (z - \alpha)^k g(z)$ where g(z) is analytic at point $z = \alpha$, $g(\alpha) \neq 0$.

b) State and prove Cauchy's Residue theorem.

c) Find the residue of the function and hence evaluat e $\int_c f(z)\,dz$ where

$$f(z) = \frac{1-2z}{z(z-1)(z-2)}.$$ **[6+4+4]**

Q7. a) By Contour integration prove that $\int_0^\infty \frac{x\sin mx}{x^2+a^2}dx = \frac{\pi}{2e^{ma}}$.

b) State and prove Argument principle.

c) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circle $|z| = 1$ and $|z| = 2$ **[5+5+4]**

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KLE Society's

Raja Lakhamagouda Science Institute (Autonomous),

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Second Semester M.Sc. Degree Examination May/June – 2015

BM06: Computer Oriented Numerical Analysis

Duration: 3 Hrs **Max Marks: 70**

Instructions to candidates:

Attempt any five questions including Q. No. 9, taking into account the internal options. Marks are shown on the right.

Important Note: 1. On completing answers, compulsorily draw diagonal lines on the remaining blank pages.
2. Any revealing of identification, appeal to valuator and / or equations written will be treated as malpractice.

1. **a)** Write a program to find the sum of two matrices using two dimensional array.
b) Define one dimensional array. Explain the initialization and memory representation of one dimensional array. **[7+7]**

2. **a)** Explain the following with syntax and example.
i) strlen() ii) strcat() iii) strcmp() iv) gets() v) puts()
b) Write a program to count the number of vowels and consonants in a given string. **[10+4]**

3. **a)** Explain calling and called function with suitable example.
b) Describe function with argument and with return values with an example.
c) Define recursion. Write a program to find the factorial of given number using recursion function. **[4+5+5]**

4. **a)** Give any five differences between structure and union.
b) Write a program to display the transpose of matrix using an array.
c) Write a note on nested function. **[5+5+4]**

5. **a)** What is pointer? Write the representation of * and & operator. Explain initialization of pointer variable.
b) With proper example, Describe call by value and call by reference in pointer.
c) Illustrate local and global variable with an example. **[5+5+4]**

6. **a)** Mention the various differences between array and structure.
b) Write a program to determine the entered string is palindrome or not?
c) Describe the concept of declaring array in structure with an example. **[4+5+5]**

7. **a)** Give the differences between printf() and gets() function.
b) Write a note on nested structure.
c) Write a program to find the product of two matrices using two dimensional array. **[3+4+7]**